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Contact correlations of charged fermions in two dimensions and the electron–hole plasma lifetime in semiconductor quantum wells

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Abstract. We consider a multicomponent ensemble of charged fermions which are constrained to move on the plane. By just retaining particle–particle ladder diagrams in Goldstone's expression for the energy shift and approximating the kernel of the resultant integral equation, we obtain for this system an analytical Yasuhara-like formula for the contact pair correlations that includes screening effects. An undesirable pseudo-screening due to the kernel approximation is overcome by adding a corrective term into the integral equation. In particular, we use the case in which the system has only two species of opposite charge to model a quantum well as a two-dimensional electron–hole plasma. The electron–hole correlation at contact so calculated is taken as the enhancement factor in the photoluminescence from the quantum well and it is checked against available experimental data for the density dependence of the electron–hole plasma lifetime.

1. Introduction

Progress in the fabrication of semiconductor heterostructures [1–5] has motivated, in recent years, an increasing interest in the study of their physical properties. Special attention has been given to the superlattices called quantum wells [6–8] which are made in such a way that carriers are limited to moving practically on a plane. The behaviour of these quasi-two-dimensional systems has been intensively investigated [9–11]. In particular, diverse photoluminescence experiments have been performed in order to achieve a better understanding of them [12, 13].

For many years several of the properties of conductor solids have been studied theoretically following the Sommerfeld point of view [14]. In that picture, the conduction electrons are considered as an ensemble of negatively charged fermions whereas the ionic lattice is thought of as a neutralizing continuum which is taken into account just through a dielectric constant (the jellium model). A similar model has been sometimes used to describe photoexcited semiconductors, the carriers being now electrons and holes [15].

Application of many-body theories to these models in three dimensions has shown that perturbation schemes, for example the random-phase approximation (RPA), describe the long-range correlations well [16, 17]. However, the RPA, like more elaborate theories which are based on self-consistent approaches (e.g. Hubbard's [18]), generally give unphysical short-range behaviour [17, 19].

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In order to handle these correlations, several authors [20–23], have used the *ladder* approximation. By considering only the ladder diagrams in the Goldstone formula for the energy shift [16], Yasuhara [21–23] found a closed expression for the contact pair correlation between electrons in the degenerate 3D jellium at metallic densities. Recently we have generalized this result to 3D multicomponent fermionic systems whose species have arbitrary masses, charges and densities [24, 25].

The ladder approximation was also applied by Isawa and Yasuhara [26] and by Nagano, Singwi and Ohnishi [27] to study short-range correlations in the degenerate 2D electron gas. In particular, Isawa and Yasuhara wrote the electron–electron pair correlation at contact, $g_{ee}(r = 0)$, in terms of a certain function $I_{ee}(k)$ that satisfies an integral equation. Since the ladder approximation retains only diagrams that connect just pairs of particles, Isawa and Yasuhara's $g_{ee}(r = 0)$ ignores screening effects which are important even in 2D. They further obtained an analytical expression for $g_{ee}(r = 0)$ by approximating the kernel in their integral equation. This expression compares surprisingly well with the Monte Carlo simulations of Tanatar and Ceperley [28] for the 2D electron gas. However, as was already recognized by Yasuhara [21], this agreement is fortuitous since the kernel approximation that they used introduces a kind of pseudo-screening that compensates for the true screening that the ladder approximation does not take into account.

In this work we first extend the analytical expression for the contact pair correlations reported in references [26] and [27] to systems with an arbitrary number of species of charged fermions, and then improve the resulting formulae in such a way that the undesirable pseudo-screening due to the kernel approximation is eliminated whereas the effects of the true screening are introduced in a more accurate form. These effects can be important, even in 2D, especially when species with charges of opposite sign are present.

An investigation of the density dependence of the electron–hole plasma decay time in semiconductor quantum wells [29] gives us a way to experimentally check the contact pair correlations that we have obtained theoretically. In fact, if the quantum wells are viewed as an electron–hole plasma on the plane, then the inverse of the total radiative decay rate should be proportional to the contact correlation between electrons and holes [15].

In the next section we obtain the analytical contact pair correlation functions of a two-dimensional multicomponent system of interacting fermions that we are looking for. To this end, following Yasuhara, we retain only ladder diagrams in the Goldstone formula and take the large-wavenumber limit of the functional derivative, with respect to the interaction potential, of the energy shift. In our calculation we explicitly consider the static screening by introducing an appropriate momentum cut-off. The kernel of the integral equation that results from these manipulations is conveniently approximated to obtain the desired analytical expression. In order to eliminate the pseudo-screening caused by the kernel approximation, we finally add a corrective term into the integral equation. In section 3, we use the particular case of two species with opposite charges to describe quantum wells as an ensemble of electrons and holes moving on the plane. Since the electron–hole correlation at contact can be taken as the enhancement factor for the photoluminescence from semiconductor quantum wells, the theory can be checked against available experimental data.

2. The multicomponent system of charged fermions

2.1. Pair correlation functions

Let us first consider in general a system of N species of charged fermions moving on a plane of area A and which is characterized by a dielectric constant κ_0. Let Z_α be the charge (in units of the electron charge e) and denote by $n_\alpha = N_\alpha/A$ the particle number density for species α ($\alpha = 1, 2, \ldots, N$), where N_α is the number of particles α lying on A.

The system Hamiltonian is written in second quantization as

$$\hat{H} = \sum_k \sum_{\alpha=1}^{N} \epsilon_k^\alpha \hat{a}_k^{\alpha^\dagger} \hat{a}_k^\alpha + \frac{1}{2A} \sum_q \sum_{\alpha\beta} v_{\alpha\beta}(q)(\hat{n}_q^\alpha \hat{n}_{-q}^\beta - \hat{N}_\alpha \delta_{\alpha\beta}) \tag{1}$$

where $v_{\alpha\beta}(q)$ is the Coulomb interaction between particles of type α and β:

$$v_{\alpha\beta}(q) = \frac{2\pi Z_\alpha Z_\beta e^2}{\kappa_0 q} \tag{2}$$

and ϵ_k^α is the kinetic energy for particles of type α with momentum k. Denoting as m_α the mass of particles α, we have

$$\epsilon_k^\alpha = \frac{\hbar^2 k^2}{2m_\alpha}. \tag{3}$$

In equation (1), $\hat{N}_\alpha = \hat{n}_{q=0}^\alpha$ indicates the particle number operator for species α, and $\hat{n}_q^\alpha$ the particle density operator for particles of species α and momentum q:

$$\hat{n}_q^\alpha = \sum_k \hat{a}_{k+q}^{\alpha^\dagger} \hat{a}_k^\alpha. \tag{4}$$

Here $\hat{a}_k^{\alpha^\dagger}$ and $\hat{a}_k^\alpha$ are creation and annihilation operators, respectively, for species α.

Our interest lies in the short-range behaviour of the pair correlation functions $g_{\alpha\beta}(r)$. These functions are proportional to the probability density of finding a particle of species α at a distance r from a particle of species β. They are related to the partial structure factors

$$S_{\alpha\beta}(q) \equiv (n_\alpha n_\beta)^{-1/2} A^{-1} \langle \hat{n}_q^\alpha \hat{n}_{-q}^\beta \rangle$$

via Fourier transforms:

$$g_{\alpha\beta}(r) = 1 + \frac{1}{(n_\alpha n_\beta)^{1/2}} \int \frac{\mathrm{d}\vec{q}}{(2\pi)^2} \, \mathrm{e}^{\mathrm{i}\vec{q}\cdot\vec{r}} \left[S_{\alpha\beta}(q) - \delta_{\alpha\beta}\right]. \tag{5}$$

In turn, the partial structure factors are expressed as functional derivatives of the energy shift ΔE caused by the perturbative interaction:

$$S_{\alpha\beta}(q) - \delta_{\alpha\beta} = \frac{2}{(n_\alpha n_\beta)^{1/2}} \frac{\delta\, \Delta E}{\delta v_{\alpha\beta}(q)}. \tag{6}$$

2.2. Contact values of correlation functions

If we consider a paramagnetic system where the diverse species have spin 1/2, then

$$g_{\alpha\beta}(r=0) = \begin{cases} \frac{1}{2} g_{\alpha\alpha}^{\uparrow\downarrow}(r=0) & \alpha = \beta \\ g_{\alpha\beta}^{\uparrow\downarrow}(r=0) & \alpha \neq \beta. \end{cases} \tag{7}$$

That is, in order to calculate $g_{\alpha\beta}(r=0)$, only the spin-up–spin-down pair correlations are needed. To obtain them we use the Goldstone formula for the energy shift [16]. Considering just ladder diagrams [21–23], this reads for unlike spins

$$\Delta E^{\uparrow\downarrow} = \frac{1}{2} \sum_q \sum_{k_1 k_2} \sum_{\alpha\beta} v_{\alpha\beta}(q) \frac{f_\alpha(k_1)\left[1 - f_\alpha(k_1+q)\right] f_\beta(k_2) \left[1 - f_\beta(k_2 - q)\right]}{\epsilon_{k_1}^\alpha - \epsilon_{k_1+q}^\alpha + \epsilon_{k_2}^\beta - \epsilon_{k_2-q}^\beta} I_{\alpha\beta}(k_1, k_2; q) \tag{8}$$

where

$$I_{\alpha\beta}(k_1, k_2; q) = v_{\alpha\beta}(q) + \sum_{q'} \frac{v_{\alpha\beta}(|\vec{q} - \vec{q}'|)\left[1 - f_\alpha(k_1 + q')\right]\left[1 - f_\beta(k_2 - q')\right]}{\epsilon^\alpha_{k_1} - \epsilon^\alpha_{k_1+q'} + \epsilon^\beta_{k_2} - \epsilon^\beta_{k_2-q'}} \times I_{\alpha\beta}(k_1, k_2; q'). \tag{9}$$

Here $f_\alpha(k)$ denotes the Fermi distribution for fermions of species α: $f_\alpha(k) = 1 - \Theta(k - k_F^\alpha)$ with $\Theta(x)$ the Heaviside step function and $k_F^\alpha = \sqrt{2\pi n_\alpha}$ the 2D Fermi momentum.

Replacing $\Delta E^{\uparrow\downarrow}$ in equation (6) by this expression we obtain, in the short-wavelength limit $q \to \infty$,

$$S^{\uparrow\downarrow}_{\alpha\beta}(q) - \delta_{\alpha\beta} = -\frac{4v_{\alpha\beta}(q)}{(n_\alpha n_\beta)^{1/2}\epsilon_q^{\alpha\beta}} \sum_{k_1k_2} f_\alpha(k_1) f_\beta(k_2) \times \left[1 + \sum_{q'} \frac{\left[1 - f_\alpha(k_1 + q')\right]\left[1 - f_\beta(k_2 - q')\right]}{\epsilon^\alpha_{k_1} - \epsilon^\alpha_{k_1+q'} + \epsilon^\beta_{k_2} - \epsilon^\beta_{k_2-q'}} I_{\alpha\beta}(k_1, k_2; q')\right]^2 \tag{10}$$

where $\epsilon_q^{\alpha\beta} = \hbar^2 q^2/(2\mu_{\alpha\beta})$ with $\mu_{\alpha\beta}$ ($\equiv m_\alpha m_\beta/(m_\alpha + m_\beta)$) the reduced mass.

From the previous equations it can be proved that, in the ladder approximation,

$$\lim_{r\to 0} g^{\uparrow\downarrow}_{\alpha\beta}(r) = -\lim_{q\to\infty} \frac{\epsilon_q^{\alpha\beta}}{(n_\alpha n_\beta)^{1/2} v_{\alpha\beta}(q)} \left[S^{\uparrow\downarrow}_{\alpha\beta}(q) - \delta_{\alpha\beta}\right] \tag{11}$$

which is the multicomponent version of the Kimball–Niklasson relation [30, 31]. Equations (11) and (10) yield

$$g^{\uparrow\downarrow}_{\alpha\beta}(r = 0) = \frac{4}{n_\alpha n_\beta} \sum_{k_1k_2} f_\alpha(k_1) f_\beta(k_2) \times \left[1 + \sum_{q'} \frac{\left[1 - f_\alpha(k_1 + q')\right]\left[1 - f_\beta(k_2 - q')\right]}{\epsilon^\alpha_{k_1} - \epsilon^\alpha_{k_1+q'} + \epsilon^\beta_{k_2} - \epsilon^\beta_{k_2-q'}} I_{\alpha\beta}(k_1, k_2; q')\right]^2. \tag{12}$$

As we are concerned with the short-range behaviour of the correlations ($|q'| \gg k_F^\alpha$; $|q'| \gg k_F^\beta$), we can approximate equation (12) by replacing the term in brackets by its value at $k_1 = k_2 = 0$ [21–27]. Using the notation $I_{\alpha\beta}(q) \equiv I_{\alpha\beta}(0, 0; q)$, we obtain

$$g^{\uparrow\downarrow}_{\alpha\beta}(r = 0) = \left[1 - \frac{\mu_{\alpha\beta}}{\hbar^2\pi} \int \frac{I_{\alpha\beta}(q)}{q^2} \Theta(q - k_F^\alpha)\Theta(q - k_F^\beta)\, \mathrm{d}\vec{q}\right]^2 \tag{13}$$

where $I_{\alpha\beta}(q)$ satisfies the integral equation

$$I_{\alpha\beta}(q) = v_{\alpha\beta}(q) - \frac{\mu_{\alpha\beta}}{\hbar^2\pi} \int v_{\alpha\beta}(|\vec{q}' - \vec{q}|) \frac{I_{\alpha\beta}(q')}{(q')^2} \Theta(q' - k_F^\alpha)\Theta(q' - k_F^\beta)\, \mathrm{d}\vec{q}'. \tag{14}$$

In the integrands of equations (13) and (14) we make the following replacement of the product of the Heaviside step functions:

$$\Theta(q - k_F^\alpha)\Theta(q - k_F^\beta) \longrightarrow \Theta(q - k_F^{\alpha\beta})$$

where the momenta cut-offs are $k_F^{\alpha\beta} = \max(k_F^\alpha, k_F^\beta)$. We further reduce q and q' with respect to the unit $k_F^{\alpha\beta}$, and $I_{\alpha\beta}(q)$ and $v_{\alpha\beta}(q)$ with respect to the unit $v_{\alpha\beta}(k_F^{\alpha\beta})$. Thus equations (13) and (14) yield

$$g^{\uparrow\downarrow}_{\alpha\beta}(r = 0) = \left[1 - \lambda_{\alpha\beta} \int_0^\infty \frac{I_{\alpha\beta}(q)}{q} \Theta(q - 1)\, \mathrm{d}q\right]^2 \tag{15}$$

and

$$I_{\alpha\beta}(q) = \frac{1}{q} - \frac{\lambda_{\alpha\beta}}{2\pi} \int \frac{\Theta(q'-1)}{|\vec{q}\,' - \vec{q}|} \frac{I_{\alpha\beta}(q')}{(q')^2}\, \mathrm{d}\vec{q}\,' \tag{16}$$

where

$$\lambda_{\alpha\beta} = \frac{2\pi Z_\alpha Z_\beta e^2}{\kappa_0 \hbar^2 \pi} \frac{\mu_{\alpha\beta}}{k_F^{\alpha\beta}}. \tag{17}$$

The integral equation (16) has the same form as the one reported by Isawa and Yasuhara [26] for the two-dimensional electron gas. By reducing its kernel according to

$$\frac{1}{|\vec{q}\,' - \vec{q}|} \sim \begin{cases} q^{-1} & q' < q \\ (q')^{-1} & q' > q \end{cases} \tag{18}$$

they obtained an approximate solution that, when introduced into the one-component version of equation (15), gives a closed analytical expression for the contact electron–electron pair correlation function for the electron gas. The dotted curve in figure 1 represents $g_{\mathrm{ee}}(0)$ versus the parameter $r_s = [a_B\sqrt{\pi n_{\mathrm{e}}}]^{-1}$, where a_B is the Bohr radius, calculated using the Isawa and Yasuhara analytical expression [26]. The two open squares indicate the values obtained from Monte Carlo simulation in reference [28]. It should be remarked that the rather good fit is merely fortuitous, since the kernel approximation, equation (18), introduces a kind of pseudo-screening that compensates for the true screening which is ignored by the ladder approximation [21].

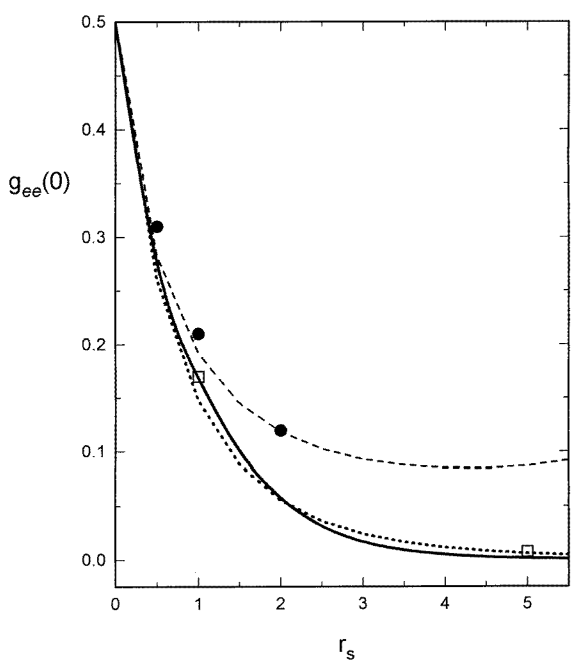

Figure 1. Contact electron–electron pair correlation functions for the 2D electron gas calculated in the ladder approximation using the Yasuhara simplification for the integral equation kernel. The solid curve includes static screening and the corrective term. The dotted curve includes neither the screening nor the corrective term. The dashed curve just includes the corrective term. The two open squares represent the corresponding results obtained from the Monte Carlo simulations of reference [32] and the full circles represent the numerical solution in the ladder approximation without screening and using the complete kernel (reference [27]).

2.3. Screening

From the previous calculations it is evident that the main interaction that the ladder approximation considers is the Coulombic one between independent pairs of particles. The interactions with all other particles are neglected, although they do reveal their presence through the existence of the Fermi surface. In particular, equation (16) does not take the static screening into account. However, we expect it to be important even in 2D, especially when charges of opposite sign coexist. Therefore it is, in principle, surprising that the formula given by Isawa and Yasuhara works so well for the electron gas. Actually, as we comment below, using the kernel approximation, equation (18), can be thought of as an indirect way of screening the interactions.

To directly account for screening effects in equation (16), we assume that the interaction between any two particles will just be effective for momentum transfers larger than some momentum cut-off k_c:

$$I_{\alpha\beta}(q) = \frac{1}{q} - \frac{\lambda_{\alpha\beta}}{2\pi}\int_{|\vec{q}\,'-\vec{q}|>k_c^{\alpha\beta}} \frac{\Theta(q'-1)}{|\vec{q}\,'-\vec{q}|}\frac{I_{\alpha\beta}(q')}{(q')^2}\,\mathrm{d}\vec{q}\,'. \tag{19}$$

As the cut-off momentum, we choose [32] $k_c = 0.43k_{TF}$ where k_{TF} is the Thomas–Fermi screening momentum. For the multicomponent system in 2D, this reads

$$k_{TF} = \frac{2}{\kappa_0 a_B}\left(\sum_{\alpha=1}^{N} m_\alpha n_\alpha Z_\alpha^2\right)\Big/\left(m_{\mathrm{e}}\sum_{\alpha=1}^{N} n_\alpha\right) \tag{20}$$

where m_{e} is the electron mass and a_B is the Bohr radius. We denote by $k_c^{\alpha\beta} = k_c/k_F^{\alpha\beta}$ the reduced form of the cut-off momentum.

We next divide the integration domain in equation (19):

$$\int_{|\vec{q}\,'-\vec{q}|>k_c^{\alpha\beta}} \cdots\,\mathrm{d}\vec{q}\,' = \int_{|\vec{q}\,'-\vec{q}|<\infty} \cdots\,\mathrm{d}\vec{q}\,' - \int_{|\vec{q}\,'-\vec{q}|<k_c^{\alpha\beta}} \cdots\,\mathrm{d}\vec{q}\,'. \tag{21}$$

Since the kernel $1/|\vec{q}\,'-\vec{q}|$ dominates at $\vec{q}\,' = \vec{q}$, the last term can be approximated [32] by evaluating $I_{\alpha\beta}(q')/(q')^2$ at that point and hence removing $I_{\alpha\beta}(q)/q^2$ from the integrand. Thus we make the approximation, for $q > k_F + k_c$,

$$\int_{|\vec{q}\,'-\vec{q}|<k_c^{\alpha\beta}} \frac{\Theta(q'-1)}{|\vec{q}\,'-\vec{q}|(q')^2} I_{\alpha\beta}(q')\,\mathrm{d}\vec{q}\,' \approx 2\pi k_c^{\alpha\beta}\frac{I_{\alpha\beta}(q)}{q^2}. \tag{22}$$

Then equation (19) yields

$$I_{\alpha\beta}(q) = \frac{1}{q} - \frac{\lambda_{\alpha\beta}}{2\pi}\int_{|\vec{q}\,'-\vec{q}|<\infty} \frac{\Theta(q'-1)}{|\vec{q}\,'-\vec{q}|}\frac{I_{\alpha\beta}(q')}{(q')^2}\,\mathrm{d}\vec{q}\,' + \lambda_{\alpha\beta}k_c^{\alpha\beta}\frac{I_{\alpha\beta}(q)}{q^2}. \tag{23}$$

In order to obtain an analytical solution, the kernel in this integral equation can be approximated by equation (18). But, as was pointed out, this approximation is equivalent to inserting an additional screening term since it underestimates the pair potential in the region $|\vec{q}\,'-\vec{q}| < k'$ with $k' \simeq 1$. Thus, the use of the estimate given by equation (18) in equation (23) implies considering a double screening. We overcome this problem by adding in equation (23) the expression

$$\begin{aligned}
&-\frac{\lambda_{\alpha\beta}}{2\pi}\int_{|\vec{q}\,'-\vec{q}|<k'} \frac{\Theta(q'-1)}{|\vec{q}\,'-\vec{q}|}\frac{I_{\alpha\beta}(q')}{(q')^2}\,\mathrm{d}\vec{q}\,' + \frac{\lambda_{\alpha\beta}}{q}\int_{q-k'}^{q}\frac{I_{\alpha\beta}(q')}{q'}\,\mathrm{d}q' + \lambda_{\alpha\beta}\int_{q}^{q+k'}\frac{I_{\alpha\beta}(q')}{(q')^2}\,\mathrm{d}q' \\
&\qquad\approx \frac{\lambda_{\alpha\beta}}{2\pi}\int_{|\vec{q}\,'-\vec{q}|<k'} \frac{\Theta(q'-1)}{|\vec{q}\,'-\vec{q}|}\frac{I_{\alpha\beta}(q')}{(q')^2}\,\mathrm{d}\vec{q}\,' \approx \lambda_{\alpha\beta}k'\frac{I_{\alpha\beta}(q)}{q^2}
\end{aligned} \tag{24}$$

so that we have

$$I_{\alpha\beta}(q) = \frac{1}{q} - \frac{\lambda_{\alpha\beta}}{q}\int_1^q \frac{I_{\alpha\beta}(q')}{q'}\,\mathrm{d}q' - \lambda_{\alpha\beta}\int_q^\infty \frac{I_{\alpha\beta}(q')}{(q')^2}\,\mathrm{d}q' + \lambda_{\alpha\beta}k_c^{\alpha\beta}\frac{I_{\alpha\beta}(q)}{q^2} - \lambda_{\alpha\beta}k'\frac{I_{\alpha\beta}(q)}{q^2}. \tag{25}$$

Using the expansion

$$I_{\alpha\beta}(q) = \sum_{n=0}^{\infty}\frac{a_n^{\alpha\beta}}{q^{n+1}} \tag{26}$$

and taking $I_{\alpha\beta}(q) \approx a_0^{\alpha\beta}/q$ in the two last terms of equation (25), we obtain

$$I_{\alpha\beta}(q) = a_0^{\alpha\beta}\left[\left(1+\frac{12k}{\lambda_{\alpha\beta}}\right)\frac{I_1(2\sqrt{\lambda_{\alpha\beta}/q})}{\sqrt{\lambda_{\alpha\beta}q}} - \frac{12k}{\lambda_{\alpha\beta}q} - \frac{6k}{q^2}\right] \tag{27}$$

where

$$a_0^{\alpha\beta} = \left[\left(1+\frac{12k}{\lambda_{\alpha\beta}}\right)I_0(2\sqrt{\lambda_{\alpha\beta}}) - 12k\left(\frac{1}{\lambda_{\alpha\beta}} + 1 + \frac{\lambda_{\alpha\beta}}{4}\right)\right]^{-1} \tag{28}$$

with I_0 and $I_1$the modified Bessel functions of the first kind and order 0 and 1, respectively, and

$$k \equiv k_c^{\alpha\beta} - k'. \tag{29}$$

Substitution of this expression for $I_{\alpha\beta}(q)$ into equation (15) finally gives for the pair correlations at zero distance

$$g_{\alpha\beta}^{\uparrow\downarrow}(r=0) = \left[1 - a_0^{\alpha\beta}\left(1+\frac{12k}{\lambda_{\alpha\beta}}\right)\left[I_0(2\sqrt{\lambda_{\alpha\beta}}) - 1\right] - 12k - 3\lambda_{\alpha\beta}k\right]^2. \tag{30}$$

We observe that for $k_c^{\alpha\beta} = k' = 0$, namely when the screening and the correction are neglected, equations (29), (30) yield

$$g_{\alpha\beta}^{\uparrow\downarrow}(r=0) = \frac{1}{\left[I_0(2\sqrt{\lambda_{\alpha\beta}})\right]^2} \tag{31}$$

which is the multicomponent version of the result that Isawa and Yasuhara reported for the two-dimensional electron gas [26].

In equations (28) and (30), k' can be considered as an adjustable parameter that depends on each particular case. However, we try to find an approximate value for it that is valid for any system. To this end, we observe that, instead of equation (24), the exact corrective term should be

$$-\frac{\lambda_{\alpha\beta}}{2\pi}\int_{|\vec{q}'-\vec{q}|<\infty}\frac{\Theta(q'-1)}{|\vec{q}'-\vec{q}|}\frac{I_{\alpha\beta}(q')}{(q')^2}\,\mathrm{d}\vec{q}' + \frac{\lambda_{\alpha\beta}}{q}\int_1^q\frac{I_{\alpha\beta}(q')}{q'}\,\mathrm{d}q' + \lambda_{\alpha\beta}\int_q^\infty\frac{I_{\alpha\beta}(q')}{(q')^2}\,\mathrm{d}q'.$$

Then, considering the expansion (26) to first order, we have that the parameter k' approximately satisfies

$$\frac{k'}{q^3} \approx \int_{|\vec{q}'-\vec{q}|<\infty}\frac{\Theta(q'-1)}{|\vec{q}'-\vec{q}|}\frac{1}{(q')^3}\,\mathrm{d}\vec{q}' - \frac{1}{q}\int_1^q\frac{1}{(q')^2}\,\mathrm{d}q' - \int_q^\infty\frac{1}{(q')^3}\,\mathrm{d}q'. \tag{32}$$

From these considerations we conclude that k' must be approximately the same for all of the systems since it does not depend on the number of species, charges and/or densities. Thus we calculate it in such a way that, when using equations (28) and (30) to describe the electron gas without screening ($k_c^{\mathrm{ee}} = 0$), we adjust one of the points that Nagano *et al* have obtained numerically using the non-approximate kernel [27]. In this way we obtain $k' = 1.3$.

In figure 1 we have plotted the contact electron–electron pair correlation as a function of r_s, calculated using equations (7) and (30), for the electron gas ($\alpha, \beta = \mathrm{e}$). We have already commented that the dotted curve is the Isawa and Yasuhara result ($k_c^{\mathrm{ee}} = k' = 0$ in equation (30), i.e. equation (31) with $\alpha, \beta = \mathrm{e}$) and that the squares are Monte Carlo results. The solid curve is obtained using the complete expression (30) with $k' = 1.3$ and $k_c^{\mathrm{ee}} = 0.608r_s$. The dashed curve, on the other hand, was obtained by setting $k_c^{\mathrm{ee}} = 0$ and $k' = 1.3$. This analytical curve neglects the static screening but includes the additional terms that correct the effect of using an approximate kernel. The full circles, finally, represent the numerical result for reference [27], which corresponds to solving the electron gas version of equations (15) and (16). We observe that the dashed curve agrees quite well with the circles. In particular, the point with $r_s = 2$ was that used to find the value of k'. It can also be appreciated that the Isawa and Yasuhara curve with neither screening ($k_c^{\mathrm{ee}} = 0$) nor correction ($k' = 0$) is in good agreement with our curve that takes into account both the static screening and the improvement to the kernel approximation. It is the fortunate effect of pseudo-screening due to the kernel approximation which explains the notable success of the Isawa–Yasuhara expression when applied to the 2D electron gas.

3. Quantum wells as an electron–hole plasma

As was mentioned, the available experimental data on luminescence from quantum wells should enable us to directly probe the contact pair correlations that we have found from the ladder approximation. In fact, these semiconductor heterostructures are fabricated in such a way that the carriers are limited to moving on very thin sheets which can, in principle, be viewed as planes.

Quantum wells have optical properties that show very interesting features due to the existence of diverse elementary excitations (excitons, biexcitons, free electron–hole (e–h) pairs etc). In GaAs/AlGaAs quantum wells, when the carrier surface densities are low ($\leqslant 5 \times 10^{10}$ cm^{-2}), the excitons determine the optical properties even at room temperature [13, 33]. At higher densities, induced by strong optical or electrical excitations, the exciton wave functions overlap, so they lose their individuality and an e–h plasma is formed [34].

The e–h plasma decay time for GaAs/$\mathrm{Ga_{0.77}Al_{0.23}As}$ quantum wells at the temperature $T = 155$ K has been measured by Bongiovanni and Staehli for diverse e–h pair densities [29]. They found that only radiative recombination is important and that the non-radiative processes have negligible effects on the total decay time. In figure 2 we show the experimental data of reference [29] which are relevant to our work. Squares show the measured total lifetimes for plasmas excited by infrared lasers. For the higher densities it is necessary to take into account that, in the experiment, once the third subbands are occupied, the absorption of the laser light decreases and the points shown are those already conveniently corrected in [29]. At densities still higher than those considered in the figure, the radiative lifetime becomes practically independent of the density due to fine intersubband transition effects [29].

In the figure, the curves for T/T_F versus the density, where $T = 155$ K and T_F is the Fermi temperature for the carriers, namely for the electrons or the holes, are also shown. For densities ranging between 5×10^{12} and 10^{13} cm^{-2}, we have that $T/T_F < 1$ for both electrons and holes, and the plasma can be considered as completely degenerate. Accordingly we argue that in that range of densities the quantum well can be viewed as a system of free electrons and holes at $T = 0$ and moving on the plane. Thus we can check the contact pair correlation that we have obtained if we consider it to be the enhancement factor in the plasma recombination rate [35]:

$$\frac{1}{\tau_r} = \frac{1}{\tau_r^0} g_{\mathrm{eh}}(0). \tag{33}$$

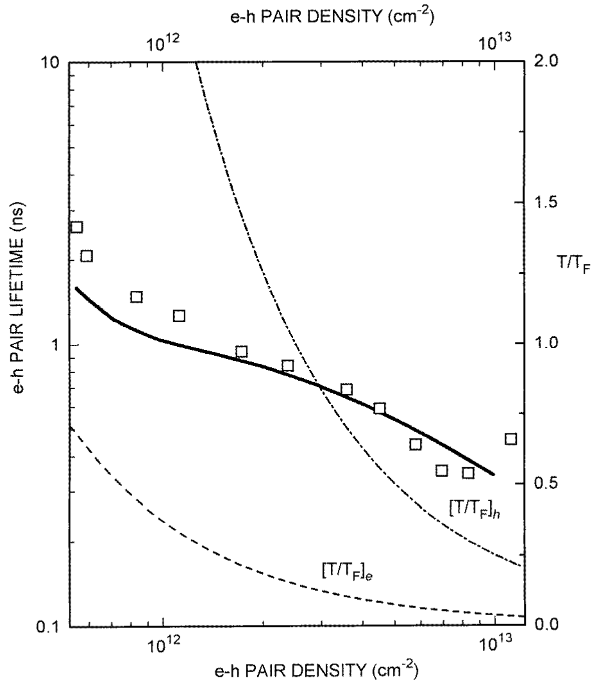

Figure 2. Left-hand axis: the electron–hole lifetime versus the electron–hole pair density for a 2D system of electrons and holes calculated in the ladder approximation using the Yasuhara simplification to the integral equation kernel and including static screening and the corrective term. The parameters in the formulae were chosen (see the text) such as to describe the GaAs/GaAlAs quantum wells of reference [29], which is also where the experimental data (open squares) were taken from. Right-hand axis: the ratio of the sample temperature (155 K) to the Fermi temperature for electrons and holes (dashed and dashed–dotted curves, respectively) versus the electron–hole pair density for the system under study.

Here τ_r and τ_r^0 denote the radiative decay time for the recombination of correlated and uncorrelated, respectively, electrons and holes.

The curve for τ_r versus the electron–hole pair density that we have calculated for the GaAs/$Ga_{0.77}Al_{0.23}$As quantum well using the equations (33), (7), (30), (28), (17) and 20), adapted for a system of just electrons and holes (with $\alpha, \beta = \mathrm{e}, \mathrm{h}$), is given by the solid curve in figure 2.

To calculate the uncorrelated decay time τ_r^0, we take into account that it is related to the exciton lifetime τ_{exc} [36, 37] according to

$$\tau_r^0 = \tau_{exc} \frac{\left|\phi_{\mathrm{eh}}^{2D}(0)\right|^2}{n} \tag{34}$$

where $n = n_\mathrm{e} = n_\mathrm{h}$ and $\phi_{\mathrm{eh}}^{2D}(0)$ denotes the contact hydrogen-like wave function describing the in-plane e–h relative motion. We have [33]: $|\phi_{\mathrm{eh}}^{2D}(0)|^2 = 2/\pi a_{exc}^2$, with $a_{exc} = a_{exc}(L_z)$ the exciton Bohr radius that depends on the thickness L_z of the quantum well. In particular, the samples used in the experiment have $L_z = 122$ Å [29].

In order to evaluate τ_r^0 using equation (34), we need to know $a_{exc}(L_z = 122$ Å) and $\tau_{exc}(T = 155$ K). We obtain the exciton radius for a GaAs sheet with $L_z = 122$ Å from the work of Bastard *et al* [38]. They plotted the dimensionless transverse extension of the exciton wave function a_{exc}/a_B versus the dimensionless well thickness L_z/a_B. The bulk GaAs exciton Bohr radius calculated with a dielectric constant $\kappa_0 = 12.96$ [39] and effective electron and holes masses $m_e^* = 0.067m_e$ and $m_h^* = 0.45m_e$ is $a_B = 118.2$ Å. Thus we obtain $a_{exc} = 130$ Å. On the other hand, from a study by Dawson *et al* [40] of the photoluminescence from GaAs/GaAlAs, 55 Å wide, for temperatures in the range 4–295 K, we obtain (by averaging the results for the two samples reported) that, at 155 K, $\tau_{exc}(L_z = 55\ \text{Å};\ T = 155\ \text{K}) \sim 8.2$ ns. Considering [38, 41] an approximate value of 1.8 for the ratio $\tau_{exc}(L_z = 122\ \text{Å};\ T = 155\ \text{K})/\tau_{exc}(L_z = 55\ \text{Å};\ T = 155\ \text{K})$, we have $\tau_{exc}(L_z = 122\ \text{Å}) = 14.8$ ns, so (with n in cm^{-2}) $\tau_r^0 \sim 5.57 \times 10^{12}/n$ ns.

According to our assumption, the enhancement factor in equation (33), $g_{\text{eh}}(0)$, is calculated for a completely degenerate system of electrons and holes using the formulae of the previous section with $k' = 1.3$. For the values of the dielectric constant and effective masses considered here, we obtain $\lambda_{\text{eh}} = -0.675 \times 10^6/\sqrt{n}$ and $k_c^{\text{eh}} = 0.306 \times 10^7/\sqrt{n}$. This value of $g_{\text{eh}}(0)$ is then used to obtain the radiative decay time for the electron–hole recombination given by the solid curve in figure 2.

We observe that, effectively, our curve fits the experimental data in the range 5×10^{12}–10^{13} cm^{-2} quite well. At lower densities, as was expected, the approximation of a degenerate plasma is not suitable. For higher densities, on the other hand, the theory is unable to account for the flattening of the e–h plasma lifetime.

4. Summary

In this work, by just summing ladder diagrams in Goldstone's perturbation formula, we have obtained a closed Yasuhara-like expression for the contact pair correlation functions of charged fermions moving on a plane which includes direct screening effects.

In order to obtain a full analytical formula, the kernel in the resultant integral equation is approximated as Isawa and Yasuhara did. This approximation is equivalent to considering an extra pseudo-screening and we improve the solution by inserting a corrective term. For the pure electron gas, the screening plus the corrective term compensate in a way that explains why the original formula of Isawa and Yasuhara for the 2D electron gas gives, by just including the pseudo-screening effect of the kernel approximation, such good results.

In the binary case we were able to check our theoretical contact pair correlation function by considering it as the plasma recombination rate enhancement factor for the photoluminescence from GaAs/GaAlAs quantum wells viewed as an electron–hole plasma. The comparison with the experimental data for the electron–hole lifetime shows good agreement over the range of densities for which a full degenerate plasma is expected.

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